The Gelfand problem for the Infinity Laplacian

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Abstract

We study the asymptotic behavior as $p \to \infty$ of solutions to the p-Laplacian Gelfand-type problem

$$\begin{cases} -\Delta_p u = \lambda\, e^u & \text{in } \Omega \subset \mathbb{R}^n \\ \quad u = 0 & \text{on } \partial\Omega. \end{cases}$$

We identify a precise scaling between u and the bifurcation parameter λ that balances reaction and diffusion and produces a nontrivial limit problem. More precisely, under an appropriate rescaling on u and λ, we prove uniform convergence of solutions to the p-Laplacian Gelfand-type problem to solutions of

$$\begin{cases} \min\{|\nabla u| - \Lambda\, e^u, -\Delta_\infty u\} = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

We discuss existence, non-existence, and multiplicity of solutions of the limit problem in terms of the limit bifurcation parameter. Moreover, we prove a comparison principle for small solutions of the limit equation. This result is interesting because the limit equation is not proper (a basic requirement for comparison) and because one cannot expect comparison to hold in general, based on the multiplicity results for the p-Laplacian Gelfand-type problem in the literature. Remarkably, minimal solutions are small in the sense of the comparison principle and we can conclude they are the only ones under a certain threshold. To the best of our knowledge, this result has no counterpart for $p < \infty$. This is a joint work with Byungjae Son (Ohio Northern University) and Pei-Yong Wang (Wayne State University).