Anomalies in Non-Linear Rayleigh Quotients

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The non-linear eigenvalue problem

$$\nabla\cdot(|\nabla u|^{p-2}\nabla u) + \lambda\,|u|^{p-2}u \;=\; 0, \quad u \in W_0^{1,p}(\Omega),$$

comes from the problem of minimizing the Rayleigh Quotient

$$\lambda(p) \;=\; \inf_{\phi} \frac{\int_\Omega |\nabla\phi|^p\,dx}{\int_\Omega |\phi|^p\,dx}$$

taken over all test functions $\phi \in C_0^\infty(\Omega)$. As a function of the parameter p, it exhibits a puzzling behaviour in *very* irregular, yet bounded, domains Ω in $\mathbb{R}^N$. I shall discuss an old counter example of mine showing that the situation

$$\lim_{s\to p-} \lambda(s) \;<\; \lambda(p), \qquad 1 < p < N,$$

is possible. A generalized Cantor Set, the Wiener Criterion, and the Kellogg Property are involved in the construction.

The phenomenon is that the "obvious" implication

$$u \in W^{1,p}(\Omega) \text{ and } u \in W_0^{1,s}(\Omega) \text{ for all } s < p \implies u \in W_0^{1,p}(\Omega)$$

is *false*.