Research statement

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Introduction

My research focus is applied mathematics and numerical analysis. Most of my work concerns modeling interaction between fluid flow and poroelastic structures. There are several phenomena in nature that involve the interaction of fluid and deformable porous material. For example, in geomechanics, the behavior of fractured aquifers and groundwater flow is significantly influenced by the interaction of deformation and flow at the pore scale, as well as the interaction between flow in the fractures and flow in the aquifer. In the same context, the efficient extraction of oil and gas from the subsurface via hydraulic fracturing relies on the ability to predict and control these phenomena. Another example from biology is the perfusion of living tissues, where the fluid carried by the main vessels is distributed by filtration to the surrounding material. All these phenomena are characterized by some common traits. In particular, they share a multiphysics nature, because they are governed by the interaction of fluid and solid mechanics. In my work, to model the free fluid I adopt the Stokes or Brinkman equation, while the flow in poroelastic media is modeled by the Biot equations. The later consists of the equations for the deformation of elastic structure and the Darcy equation that describes the average velocity of the fluid in the pores. In order to describe the interaction between two different regions, I prescribe several interface conditions on the fluid-poroelasticity interface.

The coupling between free flow and a poroelastic media has been investigated actively in recent decades due to its broad application in real life. The problem was previously analyzed from both physical and numerical points of view, using a monolithic and a partitioned approaches. In this project, I focus on the strongly coupled monolithic scheme with the approximation of the interface conditions though the use of Lagrange multipliers, which is expected to provide high accuracy among the family of methods based on weak enforcement of boundary/interface conditions.

Research progress

My research project can be divided into three phases: finished, ongoing and future work.

Finished work. I started my research with implementation of the model using the Mixed Finite Element Method (MFEM) on simplicial grids in both regions with meshes, not necessarily matching along the interface. Since the main focus was on the approximation of the interface conditions, my first project [1] was to compare the results obtained using the new method and one of the previously designed methods, in particular, the Nitsche's method, which enforces flux continuity via a penalty term [2]. We consider the test case motivated by the example investigated by Lesinigo et al[3]. The computational domain consists of two squares, representing the poroelastic material, separated by a thin fracture. Some results for the model are given on Figure 1.

In this work we verified that the model handles different material parameters and different regimes with respect to the interface friction coefficient. We also compared two different schemes, observing that they provide equivalent accuracy and precision as confirmed by the analysis of the convergence rate with respect to the mesh characteristic size. Finally, we check that the Lagrange multiplier method provides higher accuracy results for the approximation of interface conditions. Indeed, the Lagrange multiplier method is exactly enforcing the desired condition, at every

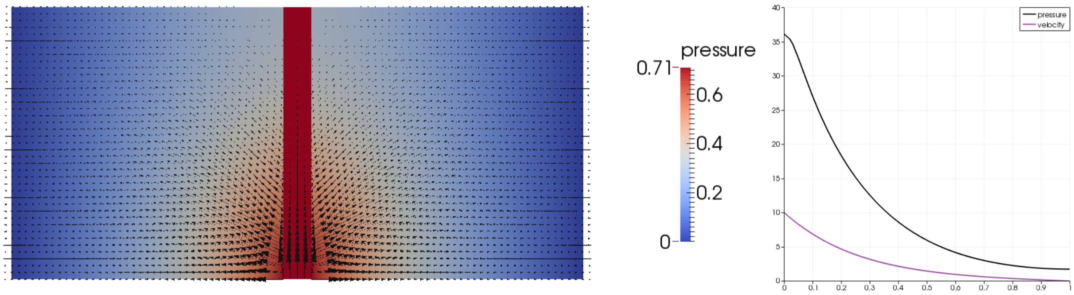
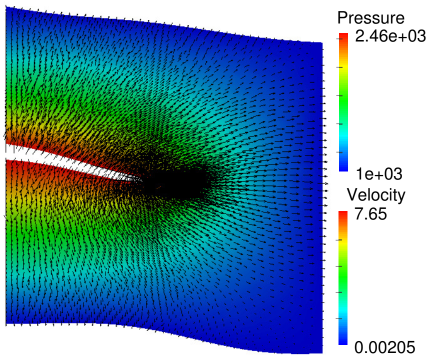


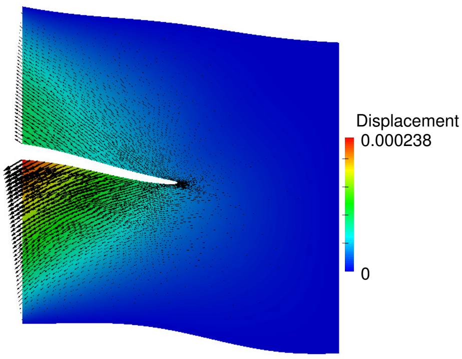
Figure 1: Velocity glyphs at the final time of simulation. For visualization purposes, the velocity field inside the fracture has been magnified of a factor 10. The background color shows the pressure magnitude (left panel). Velocity and pressure plot along the vertical meanline of the fracture region (right panel).

node on the interface, while the Nisches scheme is only approximately enforcing the condition with the corresponding residual proportionally decreasing with the mesh size.

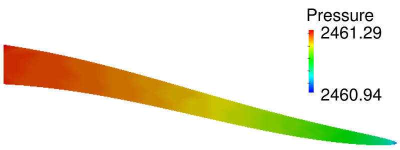
Ongoing work. The current project I'm working on [4] aims for more detailed theoretical analysis of stability and convergence of the method together with a sequence of numerical tests with irregularly shaped domains and parameters chosen to describe the real world phenomenon. Figure 2 shows the domain of interested with the solution in both regions. We have established stability of the numerical scheme and have carried out error analysis, obtaining



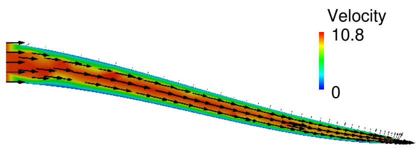
(a) Darcy velocity field over pressure in structure



(b) Structure displacement field over pressure



(c) Fluid pressure in fracture



(d) Fluid velocity field in fracture

Figure 2: Fluid flow in porous medium with homogeneous hydraulic conductivity.

optimal order convergence for all variables when stable finite elements are used in both regions. We've also tested the model in different regimes, corresponding to soft or tough rocks, that results in different behavior of the fluid inside the material. We also modeled the case of porous medium having heterogeneous parameters. In particular, we allowed the hydraulic conductivity and Young's modulus to vary within the domain. Finally, as an application to hydraulic fracturing, we also tested the injection-production case, where the injection phase involves pumping the fluid inside the medium and corresponds to the process of opening the fracture and the production phase models the opposite situations and describes the process of extraction of oil or gas.

Research during the Mellon fellowship year. The model already handles a range of parameters describing realistic properties of the porous medium. In the Mellon Fellowship year, I plan to work on the flow part of the model, so that it provides better approximation of the process of interest. Typically, the process of oil and gas extraction via hydraulic fracturing involves the flow of mixture of fluid and proppant particles, that are used to prevent the fracture from closing once the well is depressurized. Mathematically, consideration of such kind of mixture instead of just fluid, results in modeling a non-Newtonian fluid, whose viscosity is a function of the magnitude of the deformation tensor. This makes the resulting model nonlinear, adding an extra layer of complexity into the method and making both analysis and implementation much more challenging. We will perform both theoretical and numerical analysis of the modified schemes, based not only on the Lagrange multiplier method [5], but on the Nitsche's method as well, since the later will give an efficient non-iterative time splitting scheme. In particular, we will derive the results showing the well-posedness of the problem together with stability and convergence estimates. Finally, we will run various numerical experiments, illustrating the performance of the new model.

In addition to this, I will work on another scheme, which describes the flow in fracture via the dimensionally reduced Brinkman model. The latest result [6] for such a scheme guarantees only sub-optimal convergence due to the approximation of the interface terms. We will modify the model using the Lagrange multiplier method in order to obtain the optimal convergence rates.

Significance of research project

This project will advance the simulation of free flow - poroelastic medium interaction, which arises in many physical phenomena. We develop numerical approximations with good stability and convergence properties that will handle a wide range of physical parameters, with extensions to the case of heterogeneous permeability of the material and non-constant viscosity of the fluid. The resulting model will describe the process in more details and therefore lead to a deeper understanding of the physical phenomena.

References

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