RESEARCH SUMMARY

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• **Research project:** My current research project is devoted to the mathematical study of nonlinear partial differential equations arising in fluid dynamics and related areas, such as active hydrodynamics and turbulence.

• **Current research project progress:**

Firstly, we studied the 3D compressible Navier-Stokes-Korteweg (NSK) system, which describes the motions of the compressible isothermal viscous capillary fluids. In [1], by using a general energy method, we obtain the optimal time decay rates of the solutions to the NSK system, when the initial datas are small perturbations of a given constant state $(\bar{\rho}, \bar{\theta})$. In particular, the optimal decay rates of the higher-order spatial derivatives of solutions are obtained. In our proof, the negative Sobolev norms are shown to be preserved along time evolution and enhance the decay rates. Instead of using the traditional linear decay analysis, we apply a family of scaled energy estimates with minimum derivative counts and interpolations among them.

Secondly, we studied the multi-dimensional hydrodynamic equations of active nematic liquid crystals, described in the Landau-de Gennes' Q-tensor framework, both in incompressible and compressible cases. The system governs fluids with active nematic suspensions. And one of the distinctions of this active nematic model from its well-studied passive counterparts is that: it is the energy that the active suspensions consume and dissipate that drives the system out of equilibrium, rather than the external force applied at the boundary of the passive system. On account of such internal drive in the active system, the system's dynamics would be dramatically changed. Moreover, many novel effects, like the occurrence of giant density fluctuations, the spontaneous laminar flow, unconventional rheological properties, low Reynolds number turbulence, and different spatial and temporal patterns in passive systems, are disclosed from the interaction of the orientational order and the flow. Due to the physical complexity, rich phenomena, and mathematical challenges, there are not so many rigorous mathematical analysis results about the active liquid crystal system. In [2], we prove the existence of global weak solutions in three-dimensions with suitable initial datas. And by using the Littlewood-Paley decomposition, we also get the higher regularity of the weak solutions and the uniqueness of weak-strong solutions in dimension two. In [5], we proved the existence of the global weak solutions, using weak convergence methods, compactness and interpolation arguments.

Lastly, we studied the three-dimensional incompressible nonlinear system of liquid crystals (LCs), which is a simplified system to the original Ericken-Leslie's equations. In [3], we derived upper bounds for the infinite-time and space average of the L^1 norm of the Littlewood-Paley decomposition of weak solutions of the 3D periodic liquid crystal system. The result suggests that the Kolmogorov characteristic velocity scaling, holds as an upper bound for a region of wavenumbers near the dissipative cutoff.

- **Research plan for the Mellon Fellowship year:**

In the Mellon Fellowship year, I will continue my research on the active hydrodynamics and the turbulence problems. For the active hydrodynamics problems, I consider a more complicated active liquid crystal system in which the particle concentration is no longer a constant as in [2], but obeys the continuity equation. We hope to obtain the global weak solution in dimension three, and the uniqueness of strong and weak solution in dimension two. Concerning the project on the turbulence, my goal is to extend the result of [3] to the magnetohydrodynamic equations, which describes the motion of electrically conducting fluids in an electromagnetic field, with applications in a wide range of physical areas from liquid metals to cosmic plasmas.

References

[1] Z. Tan, R. Zhang, *Optimal Decay Rates of the Compressible Fluid Models of Korteweg Type*, Z. Angew. Math. Phys. 65 (2014), no. 2, 279-300.

[2] G.Q. Chen, A. Majumdar, D. Wang, R. Zhang, *Global Existence and Regularity for the Active Liquid Crystal System*, submitted.

[3] D. Wang, R. Zhang, *Universal Bounds for the Littlewood-Paley First-Order Moments of the 3D Liquid Crystal System*, preprint.

[4] D. Wang, R. Zhang, *Universal Bounds for the Littlewood-Paley First-Order Moments of the 3D Magnetohydrodynamic system*, in preparation.

[5] D. Wang, R. Zhang, *Global Weak Solution for the Compressible Active Liquid Crystal System*, preprint.